

Name: Solutions

3.1 Simplify each expression. Give each answer without negative exponents.

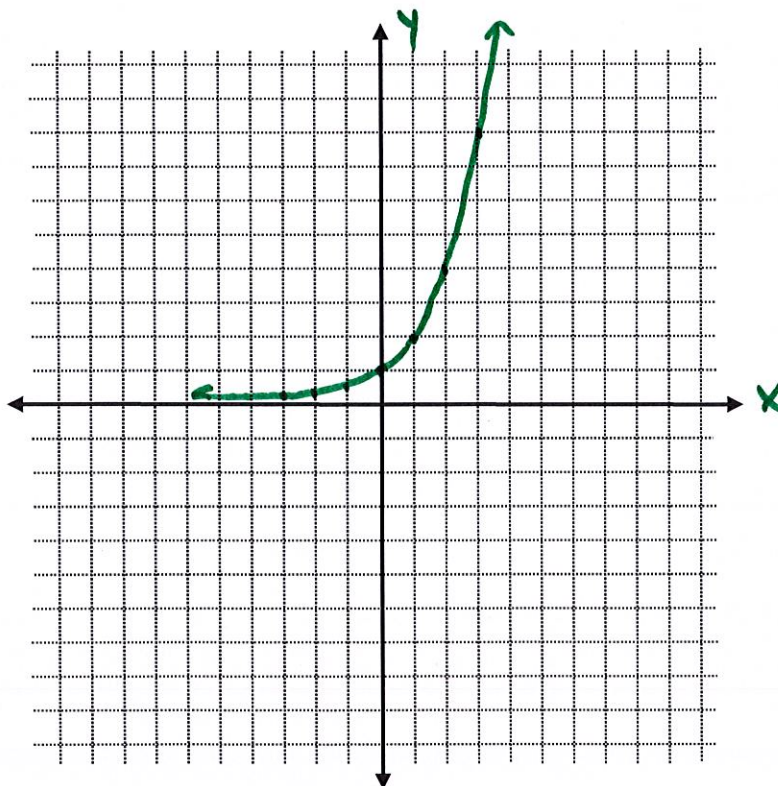
1. $x^4 + x^2 + 2x^4 + 3x^2 + x^5 =$ $x^5 + 3x^4 + 4x^2$	2. $x^4 + x^2 + y^4 + y^2 + z =$ already simplified	3. $3x^3 \cdot 5y^2 \cdot 2z =$ $30x^3y^2z$
4. $x^{11} \cdot x^2 =$ x^{13}	5. $(x^{11})^2 =$ x^{22}	6. Simplify $10^0 = 1$
7. $x^{11} \cdot x^{-2} =$ x^9	8. $\frac{x^{11}}{x^2} =$ x^9	9. $\frac{x^2}{x^{11}} =$ $\frac{1}{x^9}$
10. $\frac{x^{11}}{x^{-2}} =$ x^{13}	11. $y^{-3} =$ $\frac{1}{y^3}$	12. $\frac{1}{y^{-3}} =$ y^3
13. $17w^{-5} =$ $\frac{17}{w^5}$	14. $8^{-1}k^4 =$ $\frac{k^4}{8}$	15. $(7m^3)^2 =$ $49m^6$
16. $(3a^2)^3 =$ $27a^6$	17. $(3a^{-2})^3 =$ $27a^{-6}$ $= \frac{27}{a^6}$	18. $(3a^{-2})^{-3} =$ 3^3a^6 $= \frac{a^6}{27}$
19. $k^x \cdot k^y =$ k^{x+y}	20. $(3a^{-4})^{-3} =$ 3^3a^{12} $= \frac{a^{12}}{27}$	21. $3r \cdot 8r^3 =$ $24r^4$
22. $mn^3 \cdot 10n^2m^6 =$ $10m^7n^5$	23. $(3a^2b^5)^4 =$ $81a^8b^{20}$	24. $\frac{y^8 \cdot y^6}{y^{11}} =$ $\frac{y^{14}}{y^{11}} = y^3$
25. $\frac{y^{-8}}{y^{-11}} =$ $y^{-8} \cdot y^{11} = y^3$	26. $5k^{-5}p^7 \cdot 2^3k^8p^{-7} =$ $58k^3p^0$ $40k^3$	27. $(3v^3)^4 \cdot (5v^5)^2 \cdot v =$ $81v^{12} \cdot 25v^{10} \cdot v$ $2025v^{23}$

28. $(13t^9h^{-11})^0 = 1$	29. $\frac{y^3z}{z^5y} = \frac{y^2}{z^4}$	30. $2 \cdot 2^3 + 2 \cdot 2^{-2} = 2^4 + 2^{-1} = 16 + \frac{1}{2} = 16\frac{1}{2}$
31. Simplify $16^{\frac{1}{2}} = 4$	32. Simplify $16^{\frac{-1}{2}} = (16^{\frac{1}{2}})^{-1} = 4^{-1} = \frac{1}{4}$	33. Simplify $16^{\frac{3}{2}} = (16^{\frac{1}{2}})^3 = 4^3 = 64$
34. Simplify $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = 2^2 = 4$	35. Simplify $27^{\frac{4}{3}} = (27^{\frac{1}{3}})^4 = 3^4 = 81$	36. Simplify $10^0 = 1$
37. Simplify $10^1 = 10$	38. Simplify $10^{-1} = \frac{1}{10}$	39. Simplify $10^3 = 1,000$
40. Write $\sqrt[3]{y}$ in exponential form $y^{\frac{1}{3}}$	41. Write $\sqrt{y^5}$ in exponential form $(y^5)^{\frac{1}{2}} = y^{\frac{5}{2}}$	42. Write $\sqrt[4]{y^3}$ in exponential form $(y^3)^{\frac{1}{4}} = y^{\frac{3}{4}}$

3.2 Fill-in the y-values (outputs). Plot the points and graph the function. Some points may not fit on the grid provided.

1. $f(x) = 2^x$

X	Y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

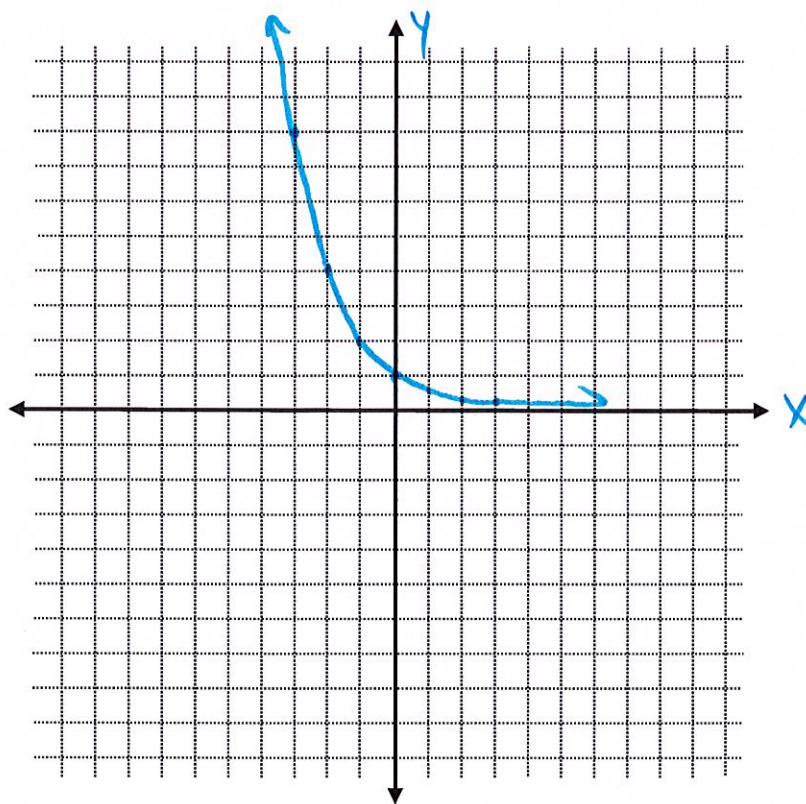


2.

Fill-in the y-values (outputs). Plot the points and graph the function. Some points may not fit on the grid provided.

2. $f(x) = \left(\frac{1}{2}\right)^x$

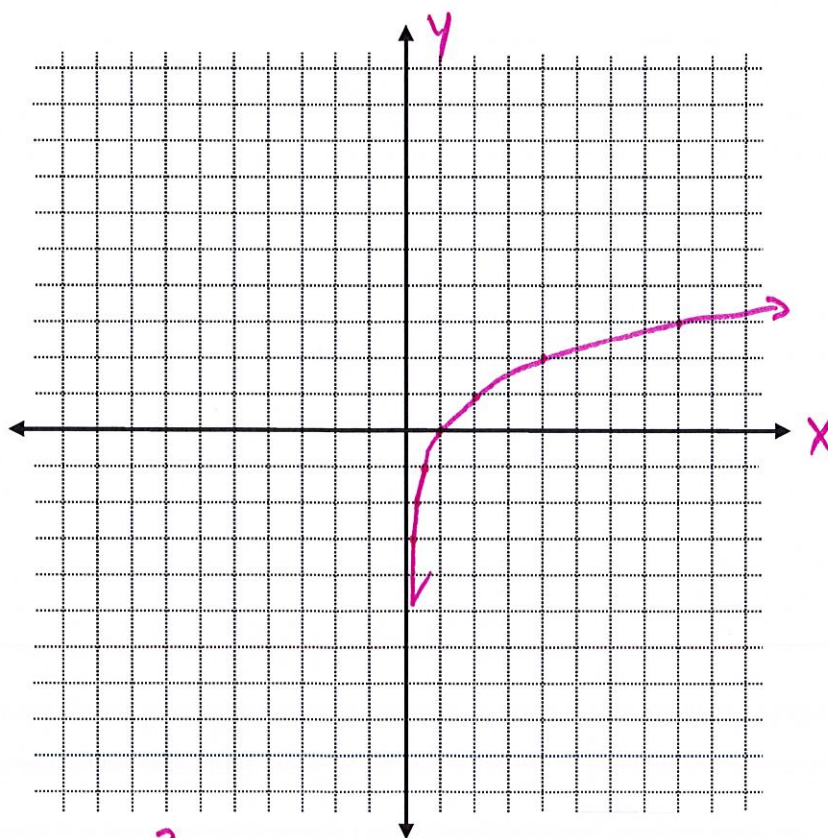
X	Y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$



Fill-in the y-values (outputs). Plot the points and graph the function. Some points may not fit on the grid provided.

3. $g(x) = \log_2 x$

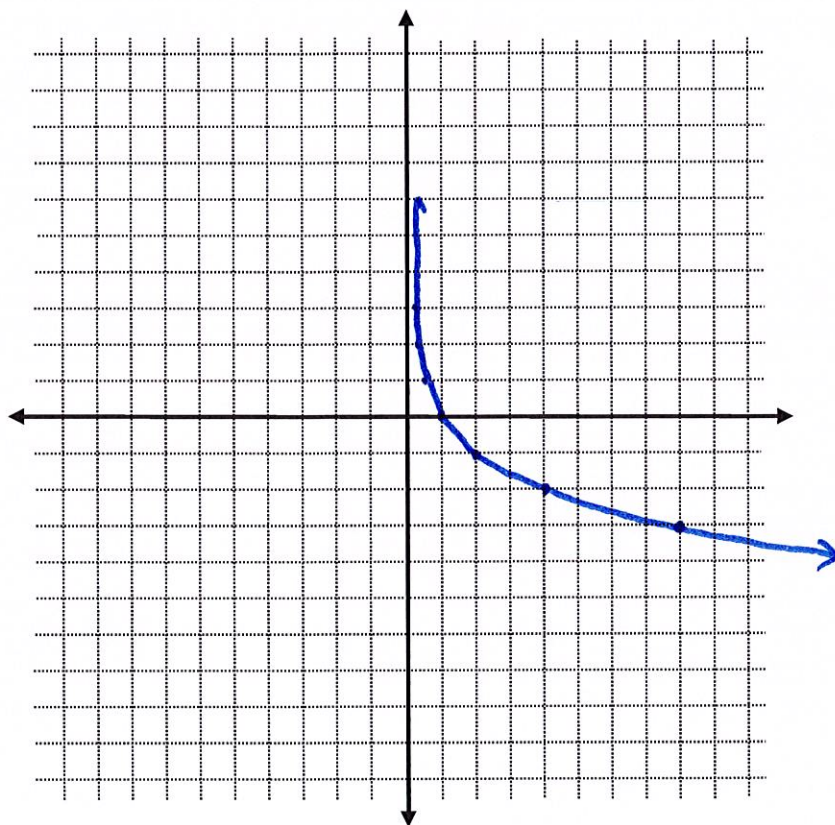
x	Y
0.125	-3
0.25	-2
0.5	-1
1	0
2	1
4	2
8	3



Fill-in the y-values (outputs). Plot the points and graph the function. Some points may not fit on the grid provided.

4. $f(x) = \log_{\frac{1}{2}} x$

X	Y
0.125	3
0.25	2
0.5	1
1	0
2	-1
4	-2
8	-3



3.3 Evaluate each logarithmic expression:

1. $\log_{10} 1000 = 3$	2. $\log_{10} 100 = 2$	3. $\log_{10} 10 = 1$
4. $\log_{10} 1 = 0$	5. $\log_{10} 0 = \text{undefined}$	6. $\log_{10} \frac{1}{10} = -1$
7. $\log_{10} \frac{1}{100} = -2$	8. $\log 1 = 0$	9. $\log 100 = 2$
10. $\log 0.1 = -1$	11. $\log(-10) = \text{undefined}$	12. $\log_2 64 = 6$
13. $\log_2 32 = 5$	14. $\log_2 16 = 4$	15. $\log_2 8 = 3$

16. $\log_2 4 = 2$	17. $\log_2 2 = 1$	18. $\log_2 1 = 0$
19. $\log_2 0 = \text{undefined}$	20. $\log_2 (-4) = \text{undefined}$	21. $\log_2 0.25 = -2$
22. $\log_2 0.5 = -1$	23. $\log_2 0.125 = -3$	24. $\log_{0.1} 100 = -2$

3.4 Use the properties of logarithms to EXPAND the following expressions (do not evaluate):

1. $\log_b (5x) = \log_b 5 + \log_b x$	2. $\log_b \left(\frac{y}{8} \right) = \log_b y - \log_b 8$
3. $\log_4 (m^5) = 5 \log_4 m$	4. $\log_5 (x^3 \cdot y) = \log_5 x^3 + \log_5 y$ $3 \log_5 x + \log_5 y$
5. $\log \left(\frac{m}{n^2} \right) = \log m - \log n^2$ $= \log m - 2 \log n$	6. $\log (a \cdot c)^4 = 4 \log (a \cdot c)$ $= 4 \log a + 4 \log c$
7. $\log (m^2 \sqrt{n}) = \log m^2 + \log \sqrt{n}$ $2 \log m + \frac{1}{2} \log n$	8. $\log \left(\frac{m}{kc} \right) = \log m - \log (kc)$ $= \log m - [\log k + \log c]$ $= \log m - \log k - \log c$

Use the properties of logarithms to CONDENSE the following expressions (do not evaluate):

9. $\log y + \log x = \log (yx)$	10. $\log_b 20 - \log_b 4 = \log_b \left(\frac{20}{4} \right) = \log_b 5$
11. $\frac{1}{2} \log 49 = \log 49^{\frac{1}{2}} = \log 7$	12. $\frac{\log 81}{4} = \frac{1}{4} \log 81 = \log 81^{\frac{1}{4}} = \log 3$
13. $5 \log_2 x + 7 \log_2 y = \log_2 x^5 + \log_2 y^7$ $= \log_2 (x^5 y^7)$	14. $3 \log_5 x - 4 \log_5 y = \log_5 x^3 - \log_5 y^4$ $= \log_5 \left(\frac{x^3}{y^4} \right)$

15. $\log A + \log B + \log C = \log(ABC)$	16. $\log A + \log B - \log C = \log(AB) - \log C = \log\left(\frac{AB}{C}\right)$
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3.5 Evaluate each logarithmic expression:

1. $\ln 7 \approx 1.9459$	2. $\log 45 \approx 1.6532$
3. If $\log_{10} 4 \approx 0.6021$, then $\log_{10} 40 \approx \log_{10}(4 \cdot 10) = \log_{10} 4 + \log_{10} 10 \approx 0.6021 + 1 \approx 1.6021$	4. If $\log 5 \approx 0.6990$, then $\log 500 \approx \log(5 \cdot 100) = \log 5 + \log 100 \approx 0.6990 + 2 \approx 2.6990$
5. If $\ln 100 \approx 4.6052$ & $\ln 4 \approx 1.3863$, then $\ln 400 \approx \ln(4 \cdot 100) = \ln 4 + \ln 100 \approx 1.3863 + 4.6052 \approx 5.9915$	6. If $\ln 5 \approx 1.6094$ & $\ln 10 \approx 2.3026$, then $\ln 500 \approx \ln(5 \cdot 100) = \ln 5 + \ln 100 \approx 1.6094 + 2.3026 = 3.9120$
7. True or False: If $\ln 2 \approx 0.6931$, then $\ln(2e^7) = \ln 2 + \ln e^7 \approx 0.6931 + 7 = 7.6931$ True	8. True or False: If $\ln 3 \approx 1.0986$, then $\ln(3e^4) = \ln 3 + \ln e^4 \approx 1.0986 + 4 = 5.0986$ True
9. $\ln(e^9) \approx 9$	10. $\log 10^7 = 7$

3.6 Solve each exponential equation WITHOUT a calculator.

1. $e^{3x} = e^{12}$ $3x = 12$ $x = 4$ $\{4\}$

2. $3^x = \frac{1}{27}$ $3^x = \frac{1}{3^3}$ $3^x = 3^{-3}$ $x = -3$ $\{-3\}$

3. $8^{x+2} = 4^{1-x}$ $2^{3(x+2)} = 2^{2(1-x)}$ $2^{3x+6} = 2^{2-2x}$ $3x+6 = 2-2x$
 $5x = -4$ $x = -\frac{4}{5}$ $\{-\frac{4}{5}\}$

4. $125^x = 25^{\frac{1}{4}x}$ $5^{3x} = 5^{2(\frac{1}{4}x)}$ $5^{3x} = 5^{\frac{1}{2}x}$ $3x = \frac{1}{2}x$ $2\frac{1}{2}x = 0$
 $x = 0$ $\{0\}$

6.

5. True or False: $4^6 = 8^4$

$$(2^2)^6 = (2^3)^4 \quad 2^{12} = 2^{12} \quad \text{True}$$

3.7 Solve each exponential equation WITH a calculator.

1. $3^x = 6$ $\log 3^x = \log 6$ $x \log 3 = \log 6$ $x = \frac{\log 6}{\log 3}$
 $x \approx 1.6309$ $\{1.6309\}$

2. $9^x = 4$ $\ln 9^x = \ln 4$ $x \ln 9 = \ln 4$ $x = \frac{\ln 4}{\ln 9}$ $x \approx 0.6309$
 $\{0.6309\}$

3. $4^{x-1} = 12$ $\log 4^{x-1} = \log 12$ $(x-1) \log 4 = \log 12$ $x-1 = \frac{\log 12}{\log 4}$
 $x \approx 1.7925 + 1 \approx 2.7925$ $\{2.7925\}$

4. $2^3 = 3^{\frac{1}{2}x}$ $\ln 8 = \ln 3^{\frac{1}{2}x}$ $\ln 8 = \frac{1}{2}x \ln 3$ $2 \ln 8 = x \ln 3$
 $\frac{2 \ln 8}{\ln 3} = x$ $x \approx 3.7856$ $\{3.7856\}$

5. $6^{x+2} + 7 = 5^2$ $6^{x+2} = 18$ $\log 6^{x+2} = \log 18$ $(x+2) \log 6 = \log 18$
 $x+2 = \frac{\log 18}{\log 6}$ $x \approx 1.6131 - 2 \approx -0.3869$ $\{-0.3869\}$

6. $3^x = e^2$ $\ln 3^x = \ln e^2$ $x \ln 3 = 2$ $x = \frac{2}{\ln 3}$ $x \approx 1.8205$ $\{1.8205\}$

3.8 Use the change of base formula to rewrite each log expression in terms of base 10. Then, use a calculator to evaluate the expression if necessary (in #'s 3-6)

1. $\log_3 x = \frac{\log x}{\log 3}$	2. $\log_x 7 = \frac{\log 7}{\log x}$
3. $\log_4 15 = \frac{\log 15}{\log 4} \approx 1.9534$	4. $\log_{15} 4 = \frac{\log 4}{\log 15} \approx 0.5119$
5. $\log_9 80 = \frac{\log 80}{\log 9} \approx 1.9943$	6. $\log_9 90 = \frac{\log 90}{\log 9} \approx 2.0480$

Use the change of base formula to rewrite each log expression in terms of base e . Then, use a calculator to evaluate the expression if necessary (in #'s 7, 8, 9, 10)

1. $\log_3 x = \frac{\ln x}{\ln 3}$	2. $\log_x 7 = \frac{\ln 7}{\ln x}$
3. $\log_4 15 = \frac{\ln 15}{\ln 4} \approx 1.9534$	4. $\log_{15} 4 = \frac{\ln 4}{\ln 15} \approx 0.5119$
5. $\log_9 80 = \frac{\ln 80}{\ln 9} \approx 1.9943$	6. $\log_9 90 = \frac{\ln 90}{\ln 9} \approx 2.0480$

3.9 Solve each equation without a calculator.

1. Solve for x : $\log(x+25) = 2$

$$10^2 = x+25 \quad 100 = x+25 \quad x = 75$$

$$\{75\}$$

2. Solve for x : $\log_{13}(x^2+25) = 2$

$$13^2 = x^2+25 \quad 169-25 = x^2$$

$$144 = x^2 \quad x = \pm 12 \quad \{12, -12\}$$

3. Solve for x : $\log_7(\log_2 x) = 1$

$$7^1 = \log_2 x \quad 7 = \log_2 x \quad 2^7 = x$$

$$x = 128 \quad \{128\}$$

4. Solve for x : $\log x = 3$

$$10^3 = x \quad 1,000 = x \quad \{1,000\}$$

5. Solve for x : $\log_x 81 = 2$

$$x^2 = 81 \quad x = \pm 9 \quad x \neq -9 \quad x = 9 \text{ only}$$

$$\{9\}$$

6. Solve for x : $\log_5 x = 4$

$$5^4 = x \quad x = 625 \quad \{625\}$$

7. Solve for y in terms of x : $2\log y = \log x$

$$\log y^2 = \log x \quad y^2 = x$$

$$y = \pm \sqrt{x} \quad \cancel{y \neq -\sqrt{x}} \quad y = \sqrt{x} \text{ only} \quad \{\sqrt{x}\}$$

8. Solve for y in terms of x : $\ln y + \ln x = \ln 90 - \ln 9$

$$\ln(yx) = \ln\left(\frac{90}{9}\right)$$

$$\ln(yx) = \ln 10 \quad yx = 10 \quad y = \frac{10}{x}$$

9. Solve for y in terms of x : $\ln\left(\frac{y}{3}\right) = x$

$$e^x = \frac{y}{3} \quad 3e^x = y$$

$$y = 3e^x$$

10. Solve for y in terms of x : $\log_x y = 3$

$$x^3 = y \quad y = x^3$$

11. Solve for x : $\log_4 2 + \log_4 (x-4) = 2$

$$\log_4 (2x-8) = 2 \quad 4^2 = 2x-8$$

$$16 = 2x-8 \quad \cancel{24 = 2x} \quad 12 = x \quad \{12\}$$

12. Solve for x : $\log_6 (x-5) + \log_6 x = 2$

$$\log_6 (x^2-5x) = 2 \quad 6^2 = x^2-5x$$

$$0 = x^2-5x-36 \quad 0 = (x-9)(x+4) \quad \begin{matrix} x \neq -4 \\ x = 9 \end{matrix}$$

$$\{9\}$$

13. Solve for x : $\ln x = -1$

$$e^{-1} = x \quad x = \frac{1}{e} \quad \left\{\frac{1}{e}\right\}$$